

Attachment 1

ADVANCED COMPOSITE MATERIALS AND PROCESSES 5.29

Since $V_f + V_m = 1$,

$$E_c = V_f E_f + E_m (1 - V_f) \quad (5.3)$$

where E_c = composite or ply Young's modulus in tension for fibers oriented in direction of applied load

V = volume fraction of fiber (f) or matrix (m)

E = Young's modulus of fiber (f) or matrix (m)

But, since the fiber has a much higher Young's modulus than the matrix ($E_f \gg E_m$) (Table 5.5 versus Table 5.7), the second part of the equation can be ignored and

$$E_c \approx E_f V_f \quad (5.4)$$

This is the basic rule of mixture and represents the highest Young's modulus composite, where all fibers are aligned in the direction of load. The minimum Young's modulus is the quasi-isotropic composite and can be approximated by

$$E_c \approx \frac{3}{8} E_f V_f \quad (5.5)$$

The quasi-isotropic modulus E of a composite laminate is $\frac{3}{8} E_{11} + \frac{3}{8} E_{22}$, where E_{22} is the transverse modulus of the lamina.¹² The transverse modulus is a small fraction of the longitudinal modulus (see E_t in Table 5.5) and can be ignored for preliminary estimates, resulting in a slightly lower than theoretical value for E_c for a quasi-isotropic laminate. This approximate value for quasi-isotropic modulus represents the lower bound of the composite modulus. It is useful for comparisons of composite properties to metal properties and to establish whether a composite is appropriate for a particular application.

The following formulas also can be used to obtain important data for unidirectional composites:

Density: $\rho_c = V_f \rho_f + V_m \rho_m \quad (5.6)$

Poisson's Ratio: $\nu_{12} = \nu_f V_f + \nu_m V_m \quad (5.7)$

Transverse Young's modulus: $E_2 = \frac{E_{2m}(1 + \xi \eta_2 V_f)}{1 - \eta_2 V_f} \quad (5.8)$

The values for η_2 and ξ can be obtained from Tsai¹³ and from Agarwal and Broutman,³ pp. 35-42. The matrix is isotropic.

Carpet Plots. The analysis of a multilayered composite if attempted by hand calculations is not trivial. Fortunately there are a significant number